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Modified gravity as an alternative for ΛCDM cosmology

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Abstract
A reconstruction scheme is developed for modified $f(R)$ gravity with realistic matter (dark matter, baryons, radiation). Two versions of such theory are constructed: the first one describes the sequence of radiation and matter domination, deceleration–acceleration transition and acceleration era, and the second one is reconstructed from exact ΛCDM cosmology. The inclusion of a radiation dominated era in the cosmological sequence is qualitatively a new result. The asymptotic behaviour of the first model at late times coincides with the theory containing positive and negative powers of curvature while the second model approaches general relativity without a singularity at zero curvature.

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1. Introduction

The modified $f(R)$ gravity (for a review, see [1]) suggests a very interesting gravitational alternative for dark energy where the current cosmic speed-up is explained by the presence of some sub-dominant terms (such as $1/R$ [2, 3] or $\ln R$ [4]) which may be caused by string/M-theory [5]. These terms may become essential at small curvature. There are simple models of modified $f(R)$ gravity such as the one with negative and positive powers of curvature [6] which are consistent with late-time astrophysical constraints [7, 8] and solar system tests [9]. The accelerating cosmology in such theories has been studied in [6, 10].

It became clear recently that modified $f(R)$ gravity may also describe the sequence of matter dominated, transition from deceleration to acceleration and acceleration eras [11, 12].

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This fact is very important because it shows that on theoretical level the modified gravity may be considered as a realistic alternative of usual general relativity where the presence of such cosmological epochs is described at any modern textbook. The reconstruction scheme which permits us to reconstruct modified gravity for any given FRW cosmology has been developed [11]. In the present paper we show that such a reconstruction scheme may be applied also in the realistic situation when usual matter is added to modified $f(R)$ gravity. Specifically, we find two implicitly given versions of $f(R)$ gravity with matter which may serve as an alternative for ΛCDM cosmology. The first model describes the sequence of radiation dominated, matter dominated and acceleration eras ,while the second model matches exactly with ΛCDM cosmology. The inclusion of a radiation dominated era to cosmological sequence in modified gravity is a qualitatively new result which was not established previously. Moreover, in the acceleration epoch the asymptotic behaviour of such theories may be defined: in the first case the model [6] containing negative power of curvature is recovered while the second model asymptotically approaches to standard general relativity. Brief arguments showing the possibility of including in the above cosmological sequence also inflationary stage are made. The fitting of the proposed modified gravity with three years WMAP data may be successfully done which shows that such a theory is indeed the alternative for usual general relativity.

2. General formulation of the reconstruction scheme

In the present section we review the reconstruction scheme for modified gravity with $f(R)$ action [11], where it has been shown how any given cosmology may define the implicit form of the function f. The starting action of modified gravity is

$$S = \int \mathrm{d}^4x\sqrt{-g}\{f(R) + \mathcal{L}_{\mathrm{matter}}\}. \tag{1}$$

Here $\mathcal{L}_{\mathrm{matter}}$ is the matter Lagrangian density. The above action is equivalently rewritten as (see, for example, [13])

$$S = \int \mathrm{d}^4x\sqrt{-g}\{P(\phi)R + Q(\phi) + \mathcal{L}_{\mathrm{matter}}\}. \tag{2}$$

Here P and Q are proper functions of the scalar field ϕ. Since the scalar field does not have a kinetic term, it may be regarded as an auxiliary field. In fact, by the variation of ϕ, it follows $0 = P'(\phi)R + Q'(\phi)$, which may be solved with respect to ϕ as $\phi = \phi(R)$. By substituting the obtained expression of $\phi(R)$ into (2), one comes back to $f(R)$-gravity:

$$S = \int \mathrm{d}^4x\sqrt{-g}\{f(R) + \mathcal{L}_{\mathrm{matter}}\}, \qquad f(R) \equiv P(\phi(R))R + Q(\phi(R)). \tag{3}$$

By the variation of the action (2) with respect to the metric $g_{\mu\nu}$, we obtain the equations corresponding to the standard spatially flat FRW universe:

$$0 = -6H^2P(\phi) - Q(\phi) - 6H\frac{\mathrm{d}P(\phi(t))}{\mathrm{d}t} + \rho, \tag{4}$$

$$0 = (4\dot{H} + 6H^2)P(\phi) + Q(\phi) + 2\frac{\mathrm{d}^2P(\phi(t))}{\mathrm{d}t} + 4H\frac{\mathrm{d}P(\phi(t))}{\mathrm{d}t} + p. \tag{5}$$

Simple algebra leads to the following equation[11]:

$$0 = 2\frac{\mathrm{d}^2P(\phi(t))}{\mathrm{d}t^2} - 2H\frac{\mathrm{d}P(\phi(t))}{\mathrm{d}\phi} + 4\dot{H}P(\phi) + p + \rho. \tag{6}$$

As one can redefine the scalar field ϕ freely, we may choose $\phi = t$. In fact, ϕ can be some function of t as $\phi = \Phi(t)$. Then we may always define a new scalar field $\hat{\phi}$ by $\phi = \Phi(\hat{\phi})$ and use $\hat{P}(\hat{\phi}) \equiv P(\Phi(\hat{\phi}))$ and $\hat{Q}(\hat{\phi}) \equiv Q(\Phi(\phi))$ instead of $P(\phi)$ and $Q(\phi)$. It is assumed that ρ and p are the sum from the contribution of the matters with a constant equation of state parameters w_i. Especially, when it is assumed to be a combination of the radiation and dust, one gets the standard expression,

$$\rho = \rho_{r0}a^{-4} + \rho_{d0}a^{-3}, \qquad p = \frac{\rho_{r0}}{3}a^{-4}, \tag{7}$$

with constant ρ_{r0} and ρ_{d0}. If the scale factor a is given by a proper function $g(t)$ as $a = a_0\,\mathrm{e}^{g(t)}$ with a constant a_0, equation (5) reduces to the second rank differential equation (see also [12]):

$$0 = 2\frac{\mathrm{d}^2 P(\phi)}{\mathrm{d}\phi^2} - 2g'(\phi)\frac{\mathrm{d}P(\phi))}{\mathrm{d}\phi} + 4g''(\phi)P(\phi) + \sum_i (1+w_i)\rho_{i0}a_0^{-3(1+w_i)}\,\mathrm{e}^{-3(1+w_i)g(\phi)}. \tag{8}$$

In principle, by solving (8) the form of $P(\phi)$ may be found. Using (4) (or equivalently (5)), the form of $Q(\phi)$ follows as

$$Q(\phi) = -6(g'(\phi))^2 P(\phi) - 6g'(\phi)\frac{\mathrm{d}P(\phi)}{\mathrm{d}\phi} + \sum_i \rho_{i0}a_0^{-3(1+w_i)}\,\mathrm{e}^{-3(1+w_i)g(\phi)}. \tag{9}$$

Hence, in principle, any given cosmology expressed as $a = a_0\,\mathrm{e}^{g(t)}$ can be realized as the solution of some specific (reconstructed) $f(R)$-gravity. Moreover, this reconstruction scheme, as shown in [14], may be generalized for other types of modified gravity.

3. Models of $f(R)$ gravity with transition of matter dominated phase to the acceleration phase

Let us consider realistic examples where the total action contains also usual matter. The starting form of $g(\phi)$ is

$$g(\phi) = h(\phi)\ln\left(\frac{\phi}{\phi_0}\right), \tag{10}$$

with a constant ϕ_0. It is assumed that $h(\phi)$ is a slowly changing function of ϕ. Due to $\phi = t$, it follows $H \sim h(t)/t$, and the effective EoS (equation of state) parameter w_{eff} is given by

$$w_{\mathrm{eff}} \equiv -1 + \frac{2\dot{H}}{H^2} \sim -1 + \frac{2}{3h(t)}. \tag{11}$$

Therefore w_{eff} changes slowly with time.

As $h(\phi)$ is assumed to be a slowly changing function of $\phi = t$, one can use adiabatic approximation and neglect the derivatives of $h(\phi)$ like $(h'(\phi) \sim h''(\phi) \sim 0)$. Then equation (8) has the following form:

$$0 = \frac{\mathrm{d}^2 P(\phi)}{\mathrm{d}\phi^2} - \frac{h(\phi)}{\phi}\frac{\mathrm{d}P(\phi))}{\mathrm{d}t} - \frac{2h(\phi)}{\phi^2}P(\phi) + \sum_i \rho_{i0}a_0^{-3(1+w_i)}\,\mathrm{e}^{-3(1+w_i)g(\phi)}. \tag{12}$$

The solution for $P(\phi)$ is found to be [11]

$$P(\phi) = p_+\phi^{n_+(\phi)} + p_-\phi^{n_-(\phi)} + \sum_i p_i(\phi)\phi^{-3(1+w_i)h(\phi)+2}. \tag{13}$$

Here $p_\pm$ are arbitrary constants and

$$n_\pm(\phi) \equiv \frac{h(\phi) - 1 \pm \sqrt{h(\phi)^2 + 6h(\phi) + 1}}{2},$$

$$p_i(\phi) \equiv -\left\{(1+w_i)\rho_{i0}a_0^{-3(1+w_i)}\phi_0^{3(1+w_i)h(\phi)}\right\} \times \{6(1+w_i)(4+3w_i)h(\phi)^2 - 2(13+9w_i)h(\phi) + 4\}^{-1}. \tag{14}$$

Especially for the radiation and dust, one has

$$p_{\mathrm{rad}}(\phi) \equiv -\frac{4\rho_{r0}\phi_0^{4h(\phi)}}{3a_0^4(40h(\phi)^2 - 32h(\phi) + 4)}, \qquad p_{\mathrm{dust}}(\phi) \equiv -\frac{\rho_{d0}\phi_0^{3h(\phi)}}{a_0^3(24h(\phi)^2 - 26h(\phi) + 4)}. \tag{15}$$

The form of $Q(\phi)$ is found to be

$$Q(\phi) = -6h(\phi)p_+(h(\phi) + n_+(\phi))\phi^{n_+(\phi)-2} - 6h(\phi)p_-(h(\phi) + n_-(\phi))\phi^{n_-(\phi)-2} + \sum_i \{-6h(\phi)(-(2+3w_i)h(\phi) + 2)p_i(\phi) + p_{i0}a_0^{-3(1+w_i)}\phi_0^{3(1+w_i)h(\phi)}\}\phi^{-3(1+w_i)h(\phi)}. \tag{16}$$

Equation (10) tells that

$$R \sim \frac{6(-h(t) + 2h(t)^2)}{t^2}. \tag{17}$$

Let us assume $\lim_{\phi\to 0} h(\phi) = h_i$ and $\lim_{\phi\to\infty} h(\phi) = h_f$. Then if $0 < h_i < 1$, the early universe is in a deceleration phase and if $h_f > 1$, the late universe is in an acceleration phase. We may consider that the case $h(\phi) \sim h_m$ is almost constant when $\phi \sim t_m$ $(0 \ll t_m \ll +\infty)$. If $h_i, h_f > 1$ and $0 < h_m < 1$, the early universe is also accelerating, which could describe the inflation. After that the universe becomes decelerating, which corresponds to the matter-dominated phase with $h(\phi) \sim 2/3$ there. Furthermore, after that, the universe enters the acceleration epoch. Hence, the unification of the inflation, matter domination and late-time acceleration is possible in the theory under consideration.

As an extension of the above model [11], we consider the inclusion of the radiation, baryons and dark matter,

$$h(\phi) = \frac{h_i + h_f q\phi^2}{1 + q\phi^2}, \tag{18}$$

with constants h_i, h_f, and q. When $\phi \to 0$, $h(\phi) \to h_i$ and when $\phi \to \infty$, $h(\phi) \to h_f$. If q is small enough, $h(\phi)$ can be a slowly varying function of ϕ. By using the expression of (17), we find [11]

$$\begin{aligned}
&\phi^2 = \Phi_0(R), \qquad \Phi_\pm(R), \\
&\Phi_0 \equiv \alpha_+^{1/3} + \alpha_-^{1/3}, \qquad \Phi_\pm \equiv \alpha_\pm^{1/3}\,\mathrm{e}^{2\pi i/3} + \alpha_\mp^{1/3}\,\mathrm{e}^{-2\pi i/3}, \\
&\alpha_\pm \equiv \frac{-\beta_0 \pm \sqrt{\beta_0^2 - \frac{4\beta_1^3}{27}}}{2}, \\
&\beta_0 \equiv \frac{2\left(2R + 6h_f q - 12h_f^2 q\right)^3}{27q^3R^3} - \frac{\left(2R + 6h_f q - 12h_f^2 q\right)(R + 6h_i q + 6h_f q - 4h_i h_f q)}{3qR} + 6h_i - 12h_i^2, \\
&\beta_1 \equiv -\frac{\left(2R + 6h_f q - 12h_f^2 q\right)^2}{3q^2R^2} - \frac{R + 6h_i q + 6h_f q - 4h_i h_f q}{q^2 R}.
\end{aligned} \tag{19}$$

There are three branches Φ_0 and $\Phi_\pm$ in (19). Equations (17) and (18) show that when the curvature is small ($\phi = t$ is large), we find $R \sim 6\left(-h_f + 2h_f^2\right)/\phi^2$ and when the curvature is large ($\phi = t$ is small), $R \sim 6\left(-h_i + 2h_i^2\right)/\phi^2$. This asymptotic behaviour indicates that we should choose Φ_0 in (19). Then the explicit form of $f(R)$ could be given by using the expressions of $P(\phi)$ (13) and $Q(\phi)$ (16) as

$$f(R) = P(\sqrt{\Phi_0(R)})R + Q(\sqrt{\Phi_0(R)}). \tag{20}$$

One may check the asymptotic behaviour of $f(R)$ (20) (for some parameters choice) in the acceleration era coincides with the theory proposed in [6]. We now consider the case where, besides dust, which could be dark matter and baryons, there is radiation. In this case, $P(\phi)$ is given by

$$P(\phi) = p_+\phi^{n_+(\phi)} + (p_{\rm dark}(\phi) + p_{\rm baryon}(\phi))\phi^{-3h(\phi)+2} + p_{\rm rad}(\phi)\,{\rm e}^{-4h(\phi)+2}. \tag{21}$$

Here $p_{\rm rad}(\phi)$ is given by (15) and $p_{\rm dark}(\phi) + p_{\rm baryon}(\phi)$ are

$$p_{\rm dark}(\phi) + p_{\rm baryon}(\phi) \equiv -\frac{(\rho_{\rm dark0} + \rho_{\rm baryon0})\phi_0^{3h(\phi)}}{a_0^3(24h(\phi)^2 - 26h(\phi) + 4)}. \tag{22}$$

We now find $n_+ > -3h(\phi) + 2 > -4h(\phi) + 2 > 0$, in (21). Here $n_+ = (h(\phi) - 1 \pm \sqrt{h(\phi)^2 + 6h(\phi) + 1})/2$ is defined in (14). Then when ϕ is large, the first term in (21) dominates and when ϕ is small, the last term dominates. When ϕ is large, the curvature is small and $\phi^2 \sim 6(-h_f + 2h_f)/R$ and $h(\phi) \to h(\infty) = h_f$. Hence, $f(R) \sim R^{-(h(\phi)-5+\sqrt{h_f^2+6h_f+1})/4}$. Especially when $h \gg 1$, it becomes $f(R) \sim R^{-h_f/2}$. Therefore there appears the negative power of R predicted by the presence of a matter-dominated stage. As $H \sim h_f/t$, if $h_f > 1$, the universe is in an acceleration phase.

On the other hand, when the curvature is large, we find $\phi^2 \sim 6(-h_i + 2h_i)/R$ and $h(\phi) \to h(0) = h_i$. If the universe era corresponds to the radiation dominated phase ($h_i = 1/2$), $P(\phi)$ becomes a constant and therefore $f(R) \sim R$, which reproduces the Einstein gravity.

Thus, in the above model, the radiation/matter dominated phase evolves into an acceleration phase and $f(R)$ behaves as $f(R) \sim R$ initially while $f(R) \sim R^{-(h(\phi)-5+\sqrt{h_f^2+6h_f+1})/4}$ at late time. For the matter dominated phase, we have $h = 2/3$. Since $h_i = 1/2 < 2/3 < h_f$ and $h(\phi = t)$ is a slowly increasing function of $\phi = t$, there should always be a matter dominated phase. Therefore in the model (18) with $h_i = 1/2$, the universe is first in the radiation dominated phase. Subsequently, the universe evolves to the matter dominated phase, and finally to the accelerated phase which is consistent with ΛCDM cosmology.

Thus, we presented the example of implicitly given $f(R)$ gravity which describes the radiation dominated era, the matter dominated stage, transition from deceleration to acceleration and acceleration epoch (where it may include the negative powers of R). This model seems to be a quite reasonable alternative for the standard ΛCDM cosmology.

4. Model reproducing ΛCDM-type cosmology

Let us investigate if ΛCDM-type cosmology could be reconstructed exactly by $f(R)$-gravity in the present formulation when we include dust, which could be a sum of the baryon and dark matter and radiation.

In the Einstein gravity, when there is a matter with the EOS parameter w and cosmological constant, the FRW equation has the following form:

$$\frac{3}{\kappa^2}H^2 = \rho_0 a^{-3(1+w)} + \frac{3}{\kappa^2 l^2}. \tag{23}$$

Here l is the length parameter. The solution of (23) is given by

$$a = a_0\,\mathrm{e}^{g(t)}, \qquad g(t) = \frac{2}{3(1+w)}\ln\left(\alpha\sinh\left(\frac{3(1+w)}{2l}(t-t_0)\right)\right). \tag{24}$$

Here t_0 is a constant of the integration and $\alpha^2 \equiv (1/3)\kappa^2 l^2 \rho_0 a_0^{-3(1+w)}$. Let us show how it is possible to reconstruct $f(R)$-gravity reproducing (24). When matter is included, equation (8) has the following form:

$$\begin{aligned} 0 = {} & 2\frac{\mathrm{d}^2 P(\phi)}{\mathrm{d}\phi^2} - \frac{2}{l}\coth\left(\frac{3(1+w)}{2l}(t\phi - t_0)\right)\frac{\mathrm{d}P(\phi)}{\mathrm{d}\phi} \\ & - \frac{6(1+w)}{l^2}\sinh^{-2}\left(\frac{3(1+w)}{2l}(\phi - t_0)\right)P(\phi) \\ & + \frac{4}{3}\rho_{r0}a_0^{-4}\left(\alpha\sinh\left(\frac{3(1+w)}{2l}(\phi - t_0)\right)\right)^{-8/3(1+w)} \\ & + \rho_{d0}a_0^{-3}\left(\alpha\sinh\left(\frac{3(1+w)}{2l}(\phi - t_0)\right)\right)^{-2/(1+w)}. \end{aligned} \tag{25}$$

Since this equation is a linear inhomogeneous equation, the general solution is given by the sum of the special solution and the general solution which corresponds to the homogeneous equation. For the case without matter, by changing the variable from ϕ to z as follows:

$$z \equiv -\sinh^{-2}\left(\frac{3(1+w)}{2l}(t - t_0)\right). \tag{26}$$

Equation (25) without matter can be rewritten in the form of Gauss's hypergeometric differential equation:

$$\begin{aligned} & 0 = z(1-z)\frac{\mathrm{d}^2 P}{\mathrm{d}z^2} + [\tilde\gamma - (\tilde\alpha + \tilde\beta + 1)z]\frac{\mathrm{d}P}{\mathrm{d}z} - \tilde\alpha\tilde\beta P, \\ & \tilde\gamma \equiv 4 + \frac{1}{3(1+w)}, \qquad \tilde\alpha + \tilde\beta + 1 \equiv 6 + \frac{1}{3(1+w)}, \qquad \tilde\alpha\tilde\beta \equiv -\frac{1}{3(1+w)}, \end{aligned} \tag{27}$$

whose solution is given by Gauss's hypergeometric function:

$$P = P_0 F(\tilde\alpha, \tilde\beta, \tilde\gamma; z) \equiv P_0 \frac{\Gamma(\tilde\gamma)}{\Gamma(\tilde\alpha)\Gamma(\tilde\beta)}\sum_{n=0}^{\infty}\frac{\Gamma(\tilde\alpha + n)\Gamma(\beta + n)}{\Gamma(\tilde\gamma + n)}\frac{z^n}{n!}. \tag{28}$$

Here Γ is the Γ-function. There is one more linearly independent solution such as $(1-z)^{\tilde\gamma - \tilde\alpha - \tilde\beta}F(\tilde\gamma - \tilde\alpha, \tilde\gamma - \tilde\beta, \tilde\gamma; z)$ but we drop it here, for simplicity. Using (9), one finds the form of $Q(\phi)$:

$$Q = -\frac{6(1-z)P_0}{l^2}F(\tilde\alpha, \tilde\beta, \tilde\gamma; z) - \frac{3(1+w)z(1-z)P_0}{l^2(13+12w)}F(\tilde\alpha + 1, \tilde\beta + 1, \tilde\gamma + 1; z). \tag{29}$$

From (26), it follows $z \to 0$ when $t = \phi \to +\infty$. Then in the limit, one arrives at $P(\phi)R + Q(\phi) \to P_0 R - 6P_0/l^2$. Identifying $P_0 = 1/2\kappa^2$ and $\Lambda = 6/l^2$, the Einstein theory with cosmological constant Λ can be reproduced. The action is not singular even in the limit of $t \to \infty$. Therefore even without cosmological constant or cold dark matter, the cosmology of ΛCDM model could be reproduced by $f(R)$-gravity (for a different treatment, see [15]).

We now investigate the special solution of (25). By changing the variable as in (26), the inhomogeneous differential equation looks as

$$\begin{aligned} & 0 = z(1-z)\frac{\mathrm{d}^2 P}{\mathrm{d}z^2} + [\tilde\gamma - (\tilde\alpha + \tilde\beta + 1)z]\frac{\mathrm{d}P}{\mathrm{d}z} - \tilde\alpha\tilde\beta P + \eta(-z)^{-2(1+3w)/3(1+w)} + \xi(-z)^{-\frac{1+2w}{1+w}}, \\ & \eta \equiv \frac{4l^2}{27(1+w)}\rho_{r0}a_0^{-4}\alpha^{-8/3(1+w)}, \qquad \zeta \equiv \frac{l^2}{3(1+w)}\rho_{d0}a_0^{-3}\alpha^{-2/(1+w)}. \end{aligned} \tag{30}$$

It is not trivial to find the solution of (30). Let us consider the case that $w = 0$ and $z \to -\infty$, that is, $t \to t_0$. In the limit, equation (30) reduces to

$$0 = -z^2 \frac{\mathrm{d}^2 P}{\mathrm{d}z^2} + \tilde{\gamma} z \frac{\mathrm{d}P}{\mathrm{d}z} - \tilde{\alpha}\tilde{\beta} P + \eta (-z)^{-2/3}, \qquad (31)$$

whose special solution is given by

$$P = P_0(-z)^{-2/3}, \qquad P_0 = \frac{\eta}{\frac{10}{9} - \frac{2(\tilde{\alpha}+\tilde{\beta}+1)}{3} + \tilde{\alpha}\tilde{\beta}} = -\frac{9\eta}{25}. \qquad (32)$$

In principle, there could be found other special solution of equation (30). This proves that even in the presence of matter, the standard ΛCDM cosmology could be reproduced by $f(R)$-gravity exactly.

Thus, we presented two versions of modified $f(R)$ gravity with matter. The first version describes the sequence of radiation dominated, matter dominated, transition from deceleration to acceleration and acceleration eras (compare with scalar-tensor gravity with the same emerging cosmology [16]). In the aceleration era the action may asymptotically approach the action with negative and positive powers of R proposed in [6]. Moreover, for some choice of parameters it reproduces the ΛCDM cosmology at late times. The second model may reproduce ΛCDM cosmology exactly. Using the number of free parameters of the models one can expect that they may be in good correspondence with observational data as they are with three years WMAP data. Nevertheless, the precise fitting of the proposed $f(R)$ gravity against existing/coming observational data should be done.

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